

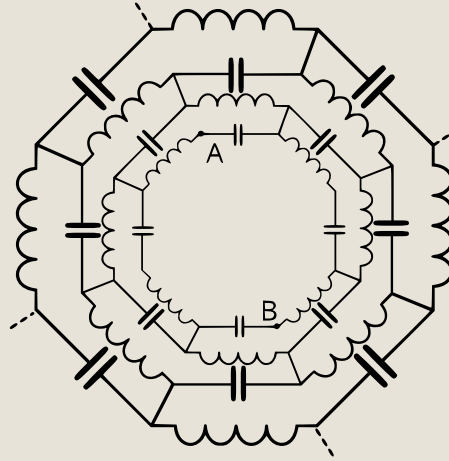
Cobweb LC network

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February 2026

Written for [Physics Cup – TalTech 2026 \(Problem 3\)](#).

At time $t = 0$, a constant voltage $V \equiv V_0$ is applied between nodes A and B of the infinite circuit shown below. All inductors have inductance L , and all capacitors have capacitance C . Find the input current $I(t)$ as a function of time.



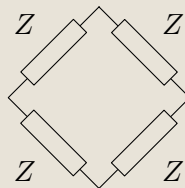
Solution

Answer:

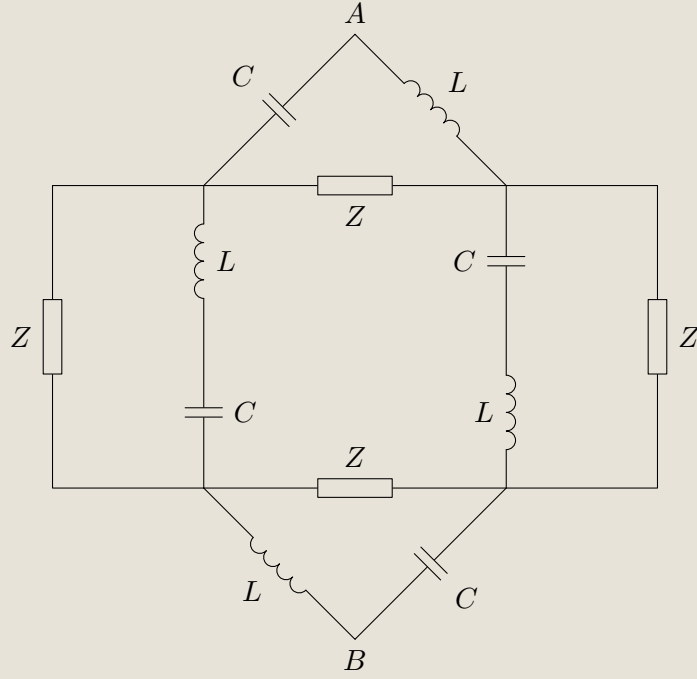
$$I(t) = V_0 \sqrt{\frac{C}{2L}}.$$

Simplifying the circuit

Let us denote the impedance between the input nodes by Z . We can see, that from the symmetry considerations and the self-similarity after removing first loop inside our circuit we can represent the rest of the circuit in terms of Z (see fig. below).



Now we can restore the original circuit as shown in figure below.

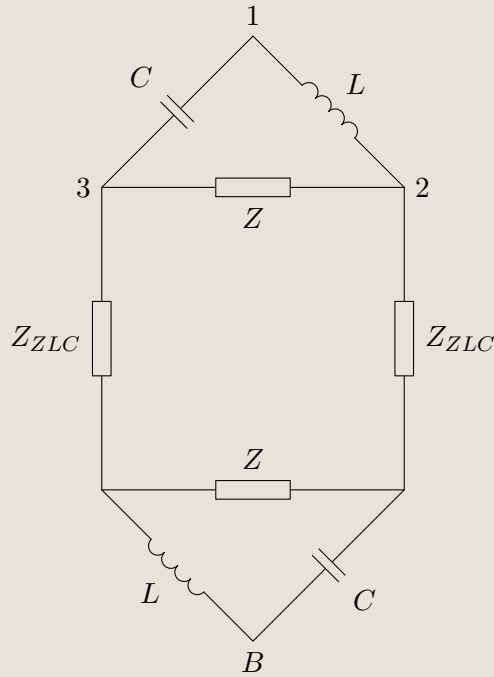


We see parallel connections $LC \parallel Z$, so that we can denote

$$Z_{ZLC} = \frac{Z \cdot Z_{LC}}{Z + Z_{LC}},$$

where $Z_{LC} = Z_L + Z_C$, Z_L and Z_C are impedances of inductors and capacitors.

Then we simplify the circuit; also we denote the node A as 1 and two neighbouring nodes of the node 1 by 2 and 3.

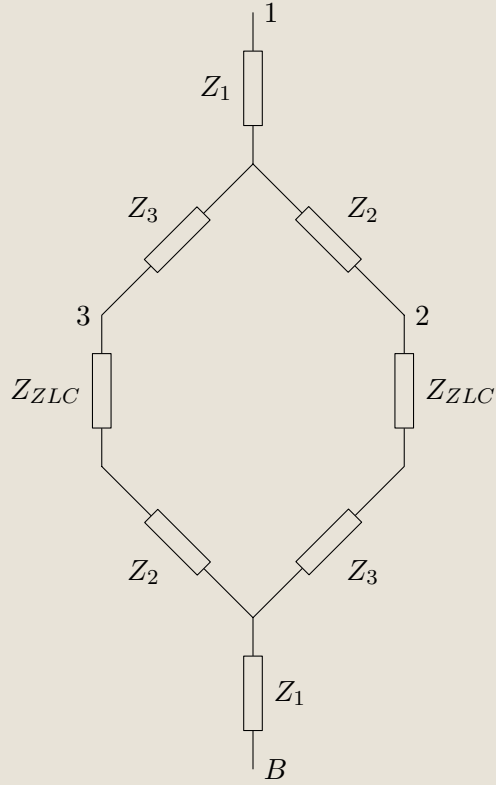


We substitute the triangular connection 1–2–3 with a star connection:

$$Z_1 = \frac{Z_C Z_L}{\Delta}, \quad Z_2 = \frac{Z_L \cdot Z}{\Delta}, \quad Z_3 = \frac{Z_C \cdot Z}{\Delta},$$

where $\Delta = Z_C + Z_L + Z$.

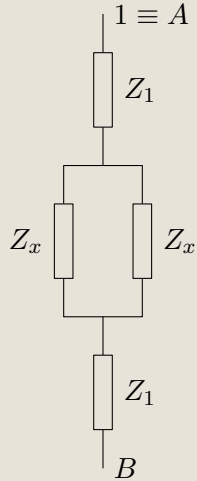
The previous circuit can be redrawn.



Then, we denote

$$Z_x = Z_2 + Z_3 + Z_{ZLC}$$

and redraw the circuit.



After this simplifying we obtain:

$$Z = 2Z_1 + \frac{Z_x}{2}.$$

Impedance calculation

In this section we will express the impedance Z in terms of Z_L and Z_C .

$$\begin{aligned} Z &= 2Z_1 + \frac{Z_x}{2} = 2Z_1 + \frac{1}{2}(Z_2 + Z_3 + Z_{ZLC}) = \\ &= 2\frac{Z_C Z_L}{\Delta} + \frac{1}{2}\left(\frac{Z_L Z}{\Delta} + \frac{Z_C Z}{\Delta} + \frac{Z Z_{LC}}{Z + Z_{LC}}\right). \end{aligned}$$

Keep in mind, that $Z + Z_{LC} = Z + Z_L + Z_C = \Delta$, therefore

$$\begin{aligned} Z &= 2\frac{Z_C Z_L}{\Delta} + \frac{Z}{2\Delta}(Z_L + Z_C + Z_{LC}) = \\ Z &= 2\frac{Z_C Z_L}{\Delta} + \frac{Z}{\Delta}(Z_L + Z_C). \end{aligned}$$

We can rewrite this

$$Z\Delta = 2Z_C Z_L + Z(Z_L + Z_C)$$

and substitute expression for Δ

$$Z(Z_C + Z_L + Z) = 2Z_C Z_L + Z(Z_L + Z_C).$$

After simplifying we obtain

$$\boxed{Z^2 = 2Z_C Z_L.}$$

Current calculation

Let us express Z in terms of L and C :

$$Z = \sqrt{2Z_L Z_C} = \sqrt{2i\omega L \frac{1}{i\omega C}} = \sqrt{2\frac{L}{C}}.$$

We see, that the impedance of the circuit between A and B is *real* number and *independent* on the angular frequency, so that the circuit can be considered as a resistor with the resistance

$$\sqrt{2\frac{L}{C}}.$$

Finally, the input current $I(t)$ is independent of time:

$$\boxed{I(t) = V_0 \sqrt{\frac{C}{2L}}.}$$